

Intermediate mass black holes - observational challenges

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Mario Pasquato
Dipartimento di Fisica E. Fermi,
Università di Pisa,
Largo Bruno Pontecorvo 3,
I-56127 Pisa, Italy

Abstract: Black holes are fascinating objects. As a class of solutions to the Einstein equations they have been studied a great deal, yielding a wealth of theoretical results. But do they really exist? What do astronomers really mean when they claim to have observational evidence of their existence? To answer these questions, I will focus on a particular range of black-hole masses, approximately from $10^2 M_\odot$ to $10^4 M_\odot$. Black holes of this size are named Intermediate Mass Black Holes (IMBHs) and their existence is still heavily disputed, so they will be perfect for illustrating the observational challenges faced by a black hole hunter.

Sažetak: Crne rupe su fascinantni objekt. One su, kao klasa rješenja Einsteinovih jednažbi, bile predmetom mnogih istraživanja koja su dovela do obilja teorijskih rezultata. No postoje li one zaista? Što astronomi točno misle kada tvrde da imaju opservacijske dokaze da one postoje? Da bih odgovorio na ova pitanja usredotočit ću se na određeni raspon mase crnih rupa, onaj između $10^2 M_\odot$ i $10^4 M_\odot$. Crne rupe ovih dimenzija nazivaju se crne rupe srednjih masa (IMBHs) i njihovo je postojanje još uvijek naširoko osporavano, tako da će one biti savršene za ilustriranje opažачkih izazova s kojima se suočava lovac na crne rupe.

Sommario: I buchi neri sono oggetti affascinanti. Come classe di soluzioni delle equazioni di Einstein, sono stati studiati approfonditamente, producendo una messe di risultati teorici. Ma esistono davvero? Che cosa intende dire un astronomo quando afferma di avere delle prove osservative della loro esistenza? Per rispondere a queste domande, mi concentrerò sull'intervallo di masse approssimativamente compreso tra $10^2 M_\odot$ e $10^4 M_\odot$. I buchi neri di queste dimensioni vengono chiamati buchi neri di massa intermedia (IMBHs) e a tutt'oggi si discute ancora se esistano o meno. Saranno quindi perfetti per illustrare le difficoltà osservative incontrate da un cacciatore di buchi neri.

1 Introduction

Intermediate Mass Black Holes (IMBHs) have masses in the range $10^2 - 10^4 M_\odot$. Stellar-mass black holes are the product of normal stellar evolution. Typically they weigh several solar masses but some have been found heavier than $10 M_\odot$ (see Creighton & Price 2008). To the best of my knowledge the record to date is $15 M_\odot$ (Bulik 2007). The so-called supermassive black holes, instead, weigh billions of solar masses and have been found in the centers of galaxy bulges, and they are thought to power much of the high-energy activity going on there. (see Kormendy & Richstone 1995; Ferrarese & Ford 2005, for reviews). IMBHs live in-between these two extremes and are actually a promising bridge between them, as they are speculated to constitute the building blocks of supermassive black holes.

Before plunging into the mysteries of IMBHs we shall deal for a while with stellar mass and supermassive BHs.

1.1 Stellar-mass black holes

The formation mechanism of stellar-mass black holes is reasonably understood, as it is based on the time-tested theory of stellar evolution. Black holes are basically what is left after stellar death. How a star dies is dependent on its mass at the time the nuclear reactions capable of sustaining it against the pull of gravity stop. During the giant-branch phase of its evolution, a star typically loses mass in variable amounts, so its final mass is not precisely constrained. Anyway, when nuclear reactions stop, the matter constituting the star collapses, unless other supporting mechanisms intervene. Electron degeneracy pressure is one of these mechanisms, and if the mass of the collapsing star is small enough it will win over gravity and the star will become a white dwarf. If electron degeneracy pressure yields to gravity, the matter is converted into neutrons and the collapse continues. Neutron degeneracy pressure is the next barrier: if the mass is not high enough, the collapse is stopped by it and a neutron star forms. If the mass is high enough, no known mechanism can stop the collapse and the star is expected to shrink until it is contained within its own Schwarzschild radius, thus becoming a black-hole. Most evidence about the existence of stellar-mass black holes comes from X-ray binaries. In these objects, an accretion disk forms and becomes hot enough to emit in the X-ray band (see McClintock & Remillard 2003). X-ray emission plays a crucial role in the context of IMBHs as well, and will be covered in more detail later.

1.2 Supermassive black holes

Supermassive black holes were first postulated as engines capable of sustaining the high-power emission of quasars (e.g. Ferrarese & Ford 2005). In the local universe they have been observed in the bulges of several galaxies either thanks to the accreting hot gas or indirectly by their effects on stellar kinematics. Also in the bulge of our own Galaxy, the Milky Way, a supermassive black hole was found. It is the supermassive black-hole with the best measurement of mass to date, as the orbits of resolved stars were used to estimate the black hole mass with $\approx 10\%$ uncertainty (Ghez et al. 2003).

1.3 Why focus on IMBHs?

We have already stated that IMBHs naturally bridge a gap between two observational regimes where relatively firm detections exist. There are further reasons that make this class of black holes interesting.

First of all, IMBHs are likely good candidates as gravitational radiation sources. If they are to be found in a large fraction of massive star clusters (see below why it is thought that star clusters might contain IMBHs), they are likely hundreds of objects in the Local Group alone. In the dense environment of a star cluster, close encounters with stellar-mass black holes and other massive objects would be quite frequent, leading to a good potential for gravitational wave emission. Estimates for LISA (Fregeau et al. 2006) or a similar space-borne interferometric detector of gravitational radiation predict tens of IMBH in-spiral events per year. Somewhat lower numbers are expected for ground based interferometers such as VIRGO or LIGO (see Miller 2002), which work in a different frequency range and would preferentially reveal black-hole merging events.

In a typical galaxy, thousands of star clusters which could potentially contain an IMBH are present. Dynamical friction due to gravitational encounters with stars could lead to an infall of star clusters (with the associated IMBHs) to the galactic bulge. Actually, so-called nuclear star clusters have recently been observed in the bulges of several galaxies. If these were to contain IMBHs they could provide a way to seed super-massive black holes, suggesting an explanation for the formation of the latter (Ebisuzaki et al. 2001; Volonteri et al. 2003; Tanaka & Haiman 2009).

More generally, IMBHs would have a strong effect on star-cluster dynamics, as they can play a crucial role in the gravothermal collapse of its core (core collapse), basically halting it. Core collapse is one of the processes resulting from the collisional evolution of clusters of stars. Stars in a star cluster evaporate as they diffuse in phase space, attaining velocities that exceed the cluster escape velocity. This way, energy is removed from the cluster, which contracts its core and moves its most massive stars to the center, in order to balance this effect (mass segregation). The three processes of evaporation, core collapse and mass segregation are all linked together in the dynamical evolution of a cluster. The amount of mass segregation attained by stars in a relaxed star cluster is also heavily dependent on the presence of an IMBH (Gill et al. 2008), which, incidentally provides a detection method (Pasquato et al. 2009).

The density and velocity dispersion cusps (Bahcall & Wolf 1976; Noyola & Gebhardt 2006, 2007) that have been observed in many star clusters could also be explained by the presence of an IMBH.

Finally, IMBHs would also naturally explain the so-called Ultra-Luminous X-ray sources (ULXs), X-ray emitting objects which appear to radiate above the Eddington limit for a stellar-mass black hole (e.g. Makishima et al. 2000).

2 IMBH formation

Several scenarios for forming black holes in the $10^2 - 10^4 M_\odot$ mass range have been proposed.

- Population III stars, i.e. the first stellar structures to form from primordial-

composition gas after the Big Bang, are predicted to be very massive. Their fast evolution can end with a black-hole remnant of several hundreds of solar masses (Madau & Rees 2001; Schneider et al. 2002),

- Simulations of dense clusters with young stars show that they may merge in a runaway cascade, producing a very massive star (Portegies Zwart et al. 2004). Such a star may produce a large black hole when it dies,
- Lower-mass black holes may merge and produce IMBHs in sufficiently high density environments. Dense star clusters (Miller & Hamilton 2002) may thus produce IMBHs, if the accreting object is not kicked out by gravitational wave recoil,
- Primordial black holes in the intermediate-mass range may form in the early evolution of the universe, according to certain variants of inflationary cosmology (Kawaguchi et al. 2008)

Two out of four mechanisms involve dense environments, so the search for IMBHs is tied to the physics of star clusters in a natural way. Two or more formation routes can be active together, leading to a composite IMBH population.

2.1 Formation from population III stars

2.1.1 Cooling and fragmentation of primordial gas

Population III stars are extremely metal poor, because they are formed from primordial composition gas. It is the physics of this gas, in particular the cooling mechanisms, which determines how discrete clumps of gas form and collapse to become population III protostars. How the gas cools sets the final mass of the clumps produced, eventually setting the mass of population III stars. Studies on this topic (Hoyle 1953; Mestel 1965a,b; Low & Lynden-Bell 1976) show that cooling is inefficient in primordial-composition gas, which implies that population III stars are characterized by much higher masses than those of stars forming in the local universe. With stellar masses as high as hundreds of solar masses, IMBHs can easily be produced as remnants.

A gas chunk of mass M undergoes gravitational collapse if the typical thermal energy kT of a particle is smaller than its binding energy:

$$kT \lesssim \frac{GMm}{R} \quad (1)$$

where R is the size of the chunk and m the particle's mass. Introducing the density $\rho \approx M/R^3$ we obtain

$$M > M_J \approx (kT/Gm)^{3/2} \rho^{-1/2} \quad (2)$$

where M_J is the Jeans mass of the gas for a given density ρ and temperature T . A gas cloud of mass $M \gg M_J$ will split into smaller fragments of mass $\approx M_J$, which collapse separately. As each chunk contracts, its density and temperature change, and

so does $M_J(T, \rho)$. Increases in M_J lead to no further fragmentation, while a decreasing M_J makes the collapse of smaller chunks possible.

If the chunk of gas is unable to efficiently cool, the collapse leads to a temperature increase. The extreme case is adiabatic collapse, where

$$T \propto \rho^{\gamma-1} \quad (3)$$

where $\gamma = c_p/c_v = 5/3$ in the case of a purely monoatomic gas. By substituting Eq. 3 into Eq. 2, we obtain

$$M_J \propto \rho^{(3\gamma-4)/2} \propto \rho^{1/2} \quad (4)$$

where the last relation holds for $\gamma = 5/3$. Density increases during collapse and M_J increases with its square root, therefore no further fragmentation takes place.

The opposite case is that of isothermal collapse, where any heat produced by collapse is immediately dissipated. Equation 2 then yields

$$M_J \propto \rho^{-1/2} \quad (5)$$

so the Jeans mass is decreasing as collapse proceeds, and further fragmentation is always possible. The more efficient the cooling mechanisms, the smaller the mass of the clumps of gas produced at the end of fragmentation.

2.1.2 Gas cooling mechanisms

Understanding the physics of gas cooling is crucial to determine the typical mass of the protostars that will form. A collapsing cloud of gas relies mainly on radiative cooling to eliminate the heat produced by the collapse. Radiative cooling depends on the radiative de-excitation of atoms and molecules excited by collisions.

Solar-composition gas contains metals, which are extremely efficient coolants. To name a few atoms¹, *C II*, *Ne II*, *Si II*, *N II*, *O II*, *O III*, have transitions which span the energy range $2.5 \times 10^{-3} \text{ eV} - 5.35 \text{ eV}$. This corresponds temperatures from $\approx 30 \text{ K}$ to $\approx 6.2 \times 10^4 \text{ K}$, which insures the gas is able to efficiently cool over a wide range of temperatures. This entails a decreasing Jeans mass until about $1 M_\odot$ clumps form (e.g. Bromm & Larson 2004). Primordial gas contains virtually no metals, and thus is must cool through less efficient channels. Also dust, which plays a role in the cooling of solar-composition gas, cannot be formed in the absence of metals. The current understanding is that molecular hydrogen, H_2 , is the main coolant in primordial gas (e.g. Bromm et al. 2002). This comparatively inefficient cooling leads to protostar masses of $\approx 100 M_\odot$ (Bromm et al. 2002; Bromm & Larson 2004; Glover 2005; Norman 2008).

2.1.3 Evolution of primordial-composition stars

As shown, population III stars are predicted to be very massive based on the physics of primordial gas cooling. Do they loose lots of mass during their evolution, or can we trust them to keep it until death, allowing the birth of an IMBH? The massive *O*-

¹In the following we make use of the spectroscopic notation, where *A I* indicates the neutral atom of element *A*, *A II* the singly ionized atom, *A III* the doubly ionized, and so on.

and *B*-type stars in the local universe lose mass through radiatively driven winds (e.g. Lucy & Solomon 1970; Castor et al. 1975; Cassinelli 1979; Abbott 1980). The force per unit mass f exerted on the gas in the outskirts of such a star depends on the opacity κ of the gas and on the radiation flux ϕ coming from the core of the star as

$$f = \frac{\kappa \Phi}{c} \quad (6)$$

The ratio f/g of radiation pressure acceleration to surface gravitational acceleration determines how much mass loss takes place. The opacity κ is an increasing function of metal content. In a population III star, opacity and thus mass-loss are expected to be much lower than in a solar-metallicity star.

Vink et al. (2001) compute the mass-loss rate of stars for various metallicities and stellar masses. They find that a $60M_{\odot}$ star with $Z = 10^{-2}Z_{\odot}$ loses about $10^{-7}M_{\odot}/yr$ via radiatively driven winds. A Population III star living for $\approx 10^7$ yr contains even less metals, leading to a cumulative mass-loss of less than $1M_{\odot}$ over its lifetime. The mass-loss is thus negligible and the mass stored in the star is fully available to produce an IMBH.

Simulations show that non-rotating population III stars with masses in the range from 50 to $140 M_{\odot}$ collapse directly to a black hole, while between 140 and $260 M_{\odot}$, explosion as a Pair-Instability SuperNova (PISN) is expected (Heger & Woosley 2002; Glover et al. 2008). Masses higher than $260 M_{\odot}$ again form a black-hole. Formation of IMBHs from population III stars predicts the existence of a *zone of avoidance* in mass corresponding to the mass range where a PISN occurs, as this kind of supernova is expected to leave no remnant.

2.2 IMBH formation from VMSs obtained by stellar merging

The runaway merging of high-mass main-sequence stars in a dense young cluster is a candidate mechanism for IMBH formation. Early on, Colgate (1967) studied the issue analytically, and in the last decades, extensive numerical simulations of young clusters have become widespread (Fregeau et al. 2006). These studies show that a Very Massive Star (VMS) forms if the lifetime of massive main-sequence stars (≈ 3 Myr) is longer than the typical time it takes for the runaway stellar merging to take place. Collisional stellar coalescence has to be fast enough to occur before the explosion of the first supernovae, which cause a sudden expansion of the cluster, reducing the number of physical collisions between stars. If a VMS is produced, its evolution likely leads to an IMBH remnant. Some high-mass stars that have been observed in the local universe, such as the Pistol star (e.g., see Figer et al. 1998) could be VMSs produced from such a stellar merging process (Freitag et al. 2006). In the following we will look more closely to the simple argument of Colgate (1967).

2.2.1 A simple model of runaway merging

Two stars of radii R_1, R_2 and of masses M_1, M_2 , with typical relative velocity at infinity V_∞ , have a cross section for physical collisions in the form:

$$\sigma = \pi(R_1 + R_2)^2 \left(1 + \frac{2G(M_1 + M_2)}{(R_1 + R_2)V_\infty^2} \right) \quad (7)$$

which differs from the purely geometrical cross-section because of the velocity-dependent expression enclosed in the rightmost bracket, which takes into account gravitational focusing. Basically, stars bend each other's orbit via gravitational attraction, resulting in an enhanced collision cross-section. The ratio between the relative velocity at infinity and the escape velocity at contact

$$V_E = \sqrt{\frac{2G(M_1 + M_2)}{(R_1 + R_2)}} \quad (8)$$

determines whether gravitational focusing is important. Eq. 7 can be rewritten as

$$\sigma = \pi(R_1 + R_2)^2 \left(1 + \frac{V_E^2}{V_\infty^2} \right) \quad (9)$$

so the two limiting cases are $V_E \ll V_\infty$, where gravitational focusing can be neglected, and $V_E \gg V_\infty$ where gravitational focusing is dominant.

For simplicity we will neglect any mass lost in stellar collisions, so all mass ends up in the star resulting from the collision. Let n be the number density of stars in the cluster and let us use the velocity dispersion $\langle v \rangle$ of cluster stars to estimate V_∞ . A given star collides with

$$\frac{dN}{dt} = n\sigma\langle v \rangle \quad (10)$$

other stars per unit time. If m is their average mass, the star's mass evolves as:

$$\frac{dM}{dt} = mn\sigma\langle v \rangle \quad (11)$$

where M is the mass of the accreting star.

The radius of a star scales approximately as $R \propto M^\beta$. Escape velocity at the star's surface scales as $V_E^2 = M/R \propto M^{1-\beta}$, so it increases with mass provided $\beta < 1$. For a massive star, gravitational focusing dominates and mass accretion is a positive feedback process. Under these assumptions, the cross section for a massive star becomes

$$\sigma \approx \pi R^2 \frac{2GM}{RV_\infty^2} \approx \frac{2\pi GM^{1+\beta}}{V_\infty^2} \quad (12)$$

where the mass (and radius) of smaller stars has been neglected. Using Eq. 11, we obtain:

$$\frac{dM}{dt} = mn \frac{2\pi GM^{1+\beta}}{V_\infty^2} \quad (13)$$

At time $t = 0$ the (soon to be a) VMS had a mass M_0 and a radius R_0 , so:

$$\frac{dM}{dt} = \frac{M_0}{\tau} \left(\frac{M}{M_0} \right)^{1+\beta} \quad (14)$$

where

$$\tau = \frac{\langle v \rangle}{2\pi G n m R_0} \quad (15)$$

so

$$M(t) = M_0 \left(1 - \frac{\beta t}{\tau} \right)^{-1/\beta} \quad (16)$$

which at time

$$t_{DIV} = \frac{\tau}{\beta} \quad (17)$$

formally diverges. A realistic value of β is $1/2$, so the divergence takes place in a finite time. For $\beta \rightarrow 0$ the growth would instead become exponential.

A VMS can thus be formed if

$$t_{DIV} < t_{MS} \approx 3 \text{ Myr} \quad (18)$$

where t_{MS} is the typical lifetime of massive main-sequence stars, i.e. the time it takes for supernova explosions to take place.

Another way to express the collisional merging timescale τ is

$$\tau = \frac{R_C}{R_0} \tau_{cr} \quad (19)$$

where R_C is the radius of the young star cluster, τ_{cr} is its crossing time and R_0 is the typical radius of a star in the cluster. This naive timescale is actually too long for runaway merging to occur in a cluster with the typical crossing time and core radius observed today. To study the infall process and the subsequent runaway merging, thus going beyond the simple model described above, numerical approaches are needed, and they must necessarily take into account core-collapse and the very high densities it attains.

2.3 IMBHs formed from merging of lower-mass black holes

Miller & Hamilton (2002) propose that dynamical interactions between binaries containing black-holes lead to repeated merging, which may result in the formation of a $\approx 10^3 M_\odot$ IMBH if the process continues for billions of years, as may be the case in old star clusters, such as galactic Globular Clusters (GCs). Three-body interactions are crucial for tightening the binaries, bringing about coalescence, but they risk to push the system out of the cluster, thus stopping the merging process. Therefore the seed black hole must be massive enough (at least $\approx 50 M_\odot$ according to Miller & Hamilton (2002)) to avoid expulsion.

In a GC, the scale binding energy per unit mass is

$$U_b = \alpha^2 \frac{GM_{GC}}{R_{GC}} \quad (20)$$

where α is a dimensionless shape factor. By setting

$$v_\infty = \sqrt{2U_b} \quad (21)$$

and, taking the typical values of a quite concentrated GC, $M_{GC} = 10^6 M_\odot$, $R_{GC} = 1 pc$, we obtain

$$v_\infty = \alpha \sqrt{\frac{2GM_{GC}}{R_{GC}}} = \alpha \times 92 km/s \quad (22)$$

whereas for a looser cluster, $M_{GC} = 10^5 M_\odot$, $R_{GC} = 4 pc$, and

$$v_\infty = \alpha \times 21 km/s \quad (23)$$

Thus, the typical escape velocity of a GC is about $\approx 50 km/s$.

Let us consider a BH binary, where the primary has mass M and the secondary has mass $m \ll M$. In an interaction with a third object of mass $\approx m$ the binary hardens (i.e. increases its binding energy) by

$$\Delta U = \eta \frac{m}{M} U \quad (24)$$

where η is typically ≈ 0.2 (Quinlan 1996). A fraction $\approx m/M$ of the energy ΔU lost by the orbital motion is transferred to the center-of-mass motion of the binary because of momentum conservation, so its overall kinetic energy increases by

$$\Delta T = \eta \frac{m^2}{M^2} U \quad (25)$$

To unbind the binary from the cluster, an increase of order the escape energy E_{esc} is needed

$$\Delta T = E_{esc} = \frac{1}{2} M v_{esc}^2 \quad (26)$$

so

$$\eta \frac{m^2}{M^2} U = \frac{1}{2} M v_{esc}^2 \quad (27)$$

and solving for U we obtain the minimum (in modulus) binding energy for which the binary is liable to be ejected due to a three-body interaction

$$U_{ej} = \frac{M^3 v_{esc}^2}{2m^2 \eta} \quad (28)$$

U_{ej} must now be compared to the minimum energy U_{co} for which the binary coalesces, due to gravitational wave emission. The time it takes the binary to coalesce with this mechanism is (Miller & Hamilton 2002)

$$t_c = \frac{5c^5}{128G^3 \mu M^2} (1 - e^2)^{7/2} \quad (29)$$

for a binary orbit with semi-axis a , eccentricity e and reduced mass $\mu = mM/(m+M) \approx m$. Equation 29 depends on orbital eccentricity, which varies in time, but we can take an average value $\langle e \rangle$. So, using Eq. 29, we obtain

$$U = \frac{GMm}{a} = \left[\frac{5Gc^5}{128} (1 - \langle e \rangle^2)^{7/2} \right]^{1/4} \frac{M^{1/2} m^{3/4}}{t_c^{1/4}} \quad (30)$$

and t_c must be shorter than the time t_e between two subsequent three-body encounters, otherwise the binary gets a new three-body kick before being able to merge. For $t_c = t_e$ we obtain:

$$U_{co} = \left[\frac{5Gc^5}{128} (1 - \langle e \rangle^2)^{7/2} \right]^{1/4} \frac{M^{1/2} m^{3/4}}{t_e^{1/4}} \quad (31)$$

Now, if $U_{co} < U_{ej}$, merging can take place before ejection. Thus

$$\left[\frac{5Gc^5}{128} (1 - \langle e \rangle^2)^{7/2} \right]^{1/4} \frac{M^{1/2} m^{3/4}}{t_e^{1/4}} < \frac{M^3 v_{esc}^2}{2m^2 \eta} \quad (32)$$

and by setting

$$k = 2\eta \left[\frac{5}{128} (1 - \langle e \rangle^2)^{7/2} \right]^{1/4} \quad (33)$$

we obtain

$$M > k^{2/5} \left[\frac{Gc^5 m^{11}}{t_e v_{esc}^8} \right]^{1/10} \quad (34)$$

An alternative way to write this condition is

$$M > k^{2/5} \left[\frac{r_S / \lambda_e}{(v_{esc}/c)^7} \right]^{1/10} m \quad (35)$$

where r_S is the Schwarzschild radius of the star of mass m , and the product $v_{esc} t_e$ was indicated with λ_e . Miller & Hamilton (2002) adopt $\eta \approx 0.2$ and $\langle e \rangle \approx 0.7$, obtaining $k \approx 0.1$. If $m = 10M_\odot$, $v_{esc} = 50 \text{ km/s}$ and $t_e = 10^6 \text{ yr}$, Eq. 34 becomes

$$M > 70M_\odot \quad (36)$$

This is approximately the threshold mass for IMBH growth through stellar-mass black hole merging. It is too large to come directly from stellar-mass black holes, so black-hole coalescence appears to need a seed black hole produced from some other process in order to work. On the other hand, once the accretion process is started, the IMBH grows quite fast, acquiring $\approx 10 M_\odot$ through merging events every $\approx 10^6 \text{ yr}$, which over the lifetime of a GC ($\approx 10^{10} \text{ yr}$) can make it grow to $\approx 10^5 M_\odot$ in mass, if enough stellar-mass black holes are present.

3 Observational evidence of IMBH existence

Evidence of the existence of IMBHs is still controversial. We will review two groups of phenomena: effects on the dynamics of stars in a cluster, and ultra-luminous X-ray emission. Several more phenomena are linked to IMBHs and could in principle yield a detection in the future: we sketch a discussion of these, but we do not add too much detail, as it would be premature.

3.1 Stellar dynamics in clusters

The best way to detect an IMBH in a star cluster would be to observe the orbits of stars bound to it. This would allow to measure the black hole mass, as in the case of the super-massive black hole in the center of the Milky Way (see Gillessen et al. 2009b; Ghez et al. 2008; Gillessen et al. 2009a; Ghez et al. 2005, 1998), resulting in an uncertainty of $\approx 10\%$, which is remarkably small by astrophysical standards. Sadly, no study of this kind has been completed with a positive result in any star cluster. This is understandable if we recall that the surroundings of the black hole in the galactic center were observed for more than 10 years, using state-of-the-art imaging technology (10 m telescopes with adaptive optics at KECK). An observational program of this kind on Galactic GCs, which are approximately 200, would literally require millennia. Moreover, cluster cores are extremely crowded (i.e. the images of several different stars will overlap) and the stars we need to observe are probably faint, main sequence stars, unless a red giant or an horizontal-branch star gets trapped in the IMBH potential well by chance.

The methods used to detect IMBHs must then be indirect, at least at this stage. Detection methods based on stellar dynamics can be divided between those which probe the zone of the cluster directly affected by the presence of the IMBH, known as the IMBH influence sphere, and those who do not need to. The former are more affected by crowding in the cluster core and by small number statistics, because the influence sphere usually contains few stars.

3.1.1 The influence sphere

Let the IMBH mass be M_{BH} and the cluster mass be M_{GC} . The size of the region where the IMBH strongly affects the dynamics of stars may be calculated by equating the escape velocity v_e at a distance R_{INF} from the IMBH to the velocity dispersion σ in the cluster core:

$$v_e = \sqrt{\frac{2GM_{BH}}{R_{INF}}} = \sigma \quad (37)$$

and

$$\sigma \approx \sqrt{\frac{GM_{GC}}{R_{GC}}} \quad (38)$$

where R_{GC} is a scale radius for the cluster. So, neglecting factors

$$\frac{R_{INF}}{R_{GC}} \approx \frac{M_{BH}}{M_{GC}} \quad (39)$$

for a typical globular star cluster, $R_{GC} \approx 1 \text{ pc}$. If we take $M_{BH}/M_{GC} = 10^{-2}$, then $R_{INF} = 10^{-2} \text{ pc}$. The distance of the nearest globular star clusters from the Sun is on the order of several kpc . At a distance of $\approx 1 \text{ kpc}$ the angular size of the influence sphere is $\approx 2 \text{ arcsec}$. For nearby clusters it is thus feasible to probe the influence sphere with present-day telescopes (e.g. Hubble Space Telescope). The volume of the influence sphere is

$$R_{INF}^3 \approx \left(\frac{M_{BH}}{M_{GC}}\right)^3 R_{GC}^3 \quad (40)$$

The number density of stars in the cluster core is a factor of $\eta \approx 10 \div 100$ larger than the average density, so the fraction of stars within the influence sphere is

$$\frac{N_{INF}}{N_{GC}} = \eta \left(\frac{M_{BH}}{M_{GC}} \right)^3 \quad (41)$$

and given that $N_{GC} \approx 10^5 - 10^6$ for a typical globular star cluster we obtain

$$N_{INF} = 10 - 10^2 \quad (42)$$

which is a small number if we hope to obtain a smooth density profile. This is one of the major hindrances encountered in revealing IMBHs using the dynamics of stars in the influence sphere. Still, some interesting results can be obtained, as shown in the following section.

3.1.2 Evidence within the influence sphere

Peebles (1972) and Bahcall & Wolf (1976) were amongst the first to predict the formation of density cusps in star clusters containing a central massive object. Both find a phase space distribution function of stars near the central mass in the form

$$f(r, v) = K(-E)^p \quad (43)$$

where E is the energy per unit mass (assuming that all stars have the same stars)

$$E = \frac{v^2}{2} - \frac{GM_{BH}}{r} \quad (44)$$

and r is the distance from the central object. The associated density profile is

$$n(r) = \int f d^3v = 4\pi K 2^{3/2} I_p \left(\frac{GM_{BH}}{r} \right)^{p+3/2} \quad (45)$$

where

$$I_p = \int_0^\infty (1 - x^2)^p x^2 dx \quad (46)$$

so a power-law cusp in the three-dimensional density profile is predicted, with exponent $p + 3/2$. Bahcall & Wolf (1976) finds $p = 1/4$ based on an analytic argument. This result is confirmed by simulations based on different numerical approaches (Shapiro 1985; Freitag & Benz 2002; Baumgardt et al. 2004; Preto et al. 2004; Amaro-Seoane et al. 2004). Observing a cusp in the core of a GC may thus be an indication of the presence of an IMBH, but some caveats apply. Cusps may form due to other physical processes, such as core collapse in a cluster containing stars with different masses (Trenti et al. 2010). On the other hand, a cusp may easily go undetected if the center of the cluster is incorrectly determined or if the resolution of the observations is limited. Notwithstanding these limitations Noyola & Gebhardt (2006) and Noyola & Gebhardt (2007) were able to reveal central luminosity density cusps in a fraction of the sample of GCs they observed using Hubble Space Telescope.

A better grip on the central dynamics of clusters can be obtained if spectroscopic (i.e. kinematic) data are also used. Noyola et al. (2008a) and Noyola et al. (2008b) obtained kinematic information for the inner 5 arcsec of ω Centauri using the Gemini telescopes, resulting in a claim to the detection of a $4 \times 10^4 M_\odot$ black hole in its center. The point here is that the observed stellar velocities increase near the center (i.e. a velocity dispersion cusp is observed). A central rise in the velocity dispersion argues for the presence of a dark mass which deepens the cluster potential well in the center. However, the kinematic information from spectroscopy is limited to line-of-sight velocities, which might not tell the whole story. A strong anisotropy in the central pressure tensor could account for the observed rise, as line-of-sight velocities could be much higher than velocities on the plane of the sky. Moreover, centering is still a delicate issue here, and the mechanisms which could generate a density cusp (see above) are likely to generate a velocity dispersion cusp as well.

At variance with the claim of Noyola et al. (2008a), van der Marel & Anderson (2010) find an upper limit of $1.2 \times 10^4 M_\odot$ to the mass of an IMBH in ω Centauri. A definitive solution to the puzzle has not been found yet.

3.1.3 Evidence outside the influence sphere

Evidence depending on resolving the influence sphere is limited by the low number of stars contained in it. Methods which rely on the global cluster structure to test for IMBH presence are free from these shortcomings. The density profile of a GC, either three-dimensional or sky projected, is characterized by two characteristic radii and the overall luminosity. The central part of the cluster is almost constant in density, up to a *core radius*, R_c , from which the density starts to decline. So R_c measures the size of the core of the cluster. The overall size of the cluster, instead, is characterized by R_h , the radius containing half of the cluster mass.

Trenti et al. (2007) use N-body numerical simulations to find that, regardless of the initial conditions, clusters with an IMBH of about 1% the total cluster mass and a realistic content of binary stars (about 10%) settle to a ratio of the (three-dimensional) core radius to the (three-dimensional) half-light radius of about

$$\frac{R_c}{R_h} \approx 0.3 \quad (47)$$

after becoming sufficiently dynamically old. This is ≈ 3 times as large as the ratio obtained for clusters without an IMBH. The core of a cluster containing an IMBH is larger in proportion to the overall size of the cluster.

Let us provide a justification of the above result from the physical point of view. We mentioned in the previous sections that a GC loses mass and energy as stars evaporate from it. Evaporation is due to two-body relaxation, i.e. stars exchanging momentum and energy in gravitational encounters. Two body relaxation promotes thermalization, i.e. the cluster phase space distribution function is driven towards a maxwellian over timescales of about one to tens of Gyr, depending on the cluster radius and number of stars. A maxwellian, however, has a tail with arbitrarily high velocities. The stars ending up in such a tail must be ejected from the cluster, removing energy and

mass from it. The evolution in time of the half-mass radius is influenced by this evaporation process as follows (the following analytical argument is loosely inspired by Vesperini & Chernoff (1994) and Hénon (1965)).

The potential energy of the cluster is

$$U = -\frac{1}{2} \sum_{i \neq j} \frac{G m_i m_j}{r_{ij}} \quad (48)$$

where m_i is the mass of the i -th star, and r_{ij} is the distance between the i -th and the j -th star. The virial theorem holds:

$$2T + U = 0 \quad (49)$$

so

$$E = T + U = \frac{U}{2} \quad (50)$$

We introduce an effective radius of the cluster such that

$$U = -\frac{GM^2}{R_{eff}} \quad (51)$$

where $M = \sum_i m_i$. Note that in general $R_{eff} \neq R_c \neq R_h$. So

$$R_{eff} = \frac{2M^2}{\sum_{i \neq j} \frac{m_i m_j}{r_{ij}}} = \frac{2N^2}{\sum_{i \neq j} \frac{1}{r_{ij}}} \quad (52)$$

where N is the total number of stars. The last equality holds approximately for equal-mass stars. The radius R_{eff} can be very different from R_h , as if any $r_{ij} \rightarrow 0$, the denominator diverges and $R_{eff} \rightarrow 0$. In other words, if stars generate subclusters (e.g. binary or triple systems), their binding energy becomes extremely high, giving a significant contribution to the total binding energy of the cluster. The ratio

$$\eta = \frac{R_{eff}}{R_h} \quad (53)$$

measures the amount of subclustering present in the system: the larger η , the less subclustering is present. From Eq. 51 and Eq. 50 we obtain

$$\frac{\dot{E}}{E} = 2 \frac{\dot{M}}{M} - \frac{\dot{R}_{eff}}{R_{eff}} \quad (54)$$

where the dot indicates a derivative with respect to time. Eq. 53 implies

$$\frac{\dot{R}_{eff}}{R_{eff}} = \frac{\dot{\eta}}{\eta} + \frac{\dot{R}_h}{R_h} \quad (55)$$

so

$$\frac{\dot{R}_h}{R_h} = 2 \frac{\dot{M}}{M} - \frac{\dot{E}}{E} - \frac{\dot{\eta}}{\eta} \quad (56)$$

The cluster loses mass due to evaporation, so $\dot{M}/M < 0$. It loses energy as well, but its binding energy is negative, so $\dot{E}/E > 0$. Then $2\dot{M}/M - \dot{E}/E < 0$. If no substructure forms, $\dot{\eta} = 0$ and the only way the cluster has to satisfy the virial theorem and conserve energy is to globally shrink, reducing R_h . Shrinking increases the binding energy, compensating for the energy lost to evaporation. If instead $\dot{\eta}/\eta$ is negative, i.e. substructure forms, the cluster can keep energy balance without shrinking. Lower values of η correspond to the formation of hierarchical subclusters, so binary stars form (or harden) in order to increase the cluster binding energy while allowing for a constant or even expanding R_h . This makes physical sense, as binaries can form only via three-body interactions (the third star acts as an energy sink). Three body interactions need a high density of stars to happen in significant numbers. An initial shrinkage of the cluster makes the central density rise, preparing the conditions for three-body interactions to take place and for binaries to be generated, or hardened if they are already present.

In simulations, the IMBH is usually found in a binary with a stellar-mass black-hole, so it acts as a source of energy, giving a significant contribution to the substructure term $\dot{\eta}/\eta$. Sources of energy in the form of substructure (several binaries or an high-mass binary with the IMBH as principal) can halt the collapse of the cluster and sustain a slight expansion in R_h as well as a large core with high R_c/R_h . This explains the result expressed by Eq. 47.

Observationally, R_c/R_h can be used as a very preliminary method to select GCs which are eligible as IMBH hosts, but several difficulties are present. Sky-projected quantities only are measurable in an observed GC. The half-mass radius is not directly observable, as only luminous matter can be used to compute it, and the dark remnants (neutron stars, stellar mass black holes, white dwarfs) are typically more massive than visible stars and tend to segregate to the center of GCs, thus leading to an overestimate of R_h as in general the light profile does not trace the mass profile of the cluster. This is a major hindrance, as the relative amount of stellar-mass black holes and neutron stars with respect to the visible stellar component is poorly constrained.

3.2 Ultra Luminous X-ray sources (ULXs)

3.2.1 Eddington limit

A source of radiation of mass M held together by its own gravity cannot have an arbitrarily high luminosity. A particle of mass m and charge q at distance r from the source feels gravitational attraction to the source

$$F_g = \frac{GMm}{r^2} \quad (57)$$

and repulsion from it due to radiation pressure

$$F_r = \frac{\sigma L}{4\pi cr^2} \quad (58)$$

where $\sigma = \sigma(q)$ is the cross section of the particle to the radiation from the source, and L is the source luminosity. Equation 58 holds only in the case of spherically symmetric emission.

By requiring that $F_r < F_g$ so the particle is globally attracted to the source we obtain

$$L < \frac{GMm4\pi c}{\sigma} = L_{EDD} \quad (59)$$

so for any mass M of the source there exists a critical luminosity L_{EDD} at which radiation pressure would destroy the source. We can take σ_{Te} (the Thompson cross section) as the electron cross section

$$\sigma_{Te} = \frac{8\pi}{3} r_e^2 = \frac{8\pi q_e^4}{3m_e^2 c^4} \quad (60)$$

where r_e is the *classical electron radius*, q_e and m_e the electron's charge and mass. Instead for a proton we obtain

$$\sigma_{Tp} = \frac{8\pi q_e^4}{3m_p^2 c^4}. \quad (61)$$

Thus $\sigma_{Tp} \ll \sigma_{Te}$, because $m_e \ll m_p$. As an estimate for hydrogen, we use $m_p = 1.67 \times 10^{-24} \text{ g}$ for m and $\sigma_{Te} = 6.65 \times 10^{-25} \text{ cm}^2$ for σ , obtaining

$$L_{EDD} = \frac{M}{M_\odot} \times 1.26 \times 10^{38} \text{ erg/s} \quad (62)$$

as the maximum allowed luminosity for the source not to be destroyed by its own radiation pressure.

3.2.2 Observing and explaining ULXs

An unresolved X-ray source (i.e. a source that appears point-like in observations) is defined as ultra-luminous if

$$L_X > 10^{39} \text{ erg/s} \quad (63)$$

i.e. if its luminosity in the X-ray band exceeds the Eddington luminosity of an $\approx 10M_\odot$ mass. The centers of galaxies often display X-ray activity, but they are usually excluded from this class of objects. Thus only those sources which are off-center with respect to the parent galaxy are counted among the ULXs.

High-luminosity X-ray sources began to be discovered in galaxies thanks to the *Einstein Observatory* mission in 1978 (Fabbiano 1989). ROSAT, ASCA, *Chandra* and *XMM-Newton* followed, resulting in a wealth of data on the X-ray sky. ULXs are widespread in nearby galaxies, but a satisfactory understanding of the underlying physical mechanisms has not been attained yet (Fabbiano 2006). To achieve luminosities as high as 10^{39} erg/s , the only plausible physical mechanism proposed so far is accretion of matter onto a compact object. However, by definition, ULXs are impossible to explain by isotropic, sub-Eddington emission from an accreting stellar mass black hole of $M \lesssim 10M_\odot$.

The only remaining viable explanations are that either ULXs are beamed sources or somehow achieve super-Eddington emission, or a new class of accreting objects, that is IMBHs, is the solution to this observational riddle. We point the reader to Colbert & Miller (2005) for a review of the observational evidence supporting the latter point of view. In the following we briefly review the most important piece of evidence, that is spectroscopic data.

3.2.3 Spectroscopy and the MCD model

The Multi Color Disk (MCD) model was first used to fit the X-ray spectra of Galactic sources emitting at lower luminosities than 10^{39} erg/s . It stems essentially from the superposition of a blackbody spectrum and the so-called multi color disk spectrum (Mitsuda et al. 1984; Takano et al. 1994). Galactic X-ray sources are neutron stars or stellar-mass black holes accreting mass from a companion star. The accreting matter forms a disk orbiting the neutron star or black hole and increases its temperature as it nears its surface. The blackbody component corresponds to the surface of the accreting object, and the multi color disk component is a sum of blackbody sources at different temperatures intended to model the accretion disk.

A particle of mass m in a newtonian orbit of semi-axis r around the accretor mass M has an energy

$$E = -\frac{GMm}{2r} \quad (64)$$

so, a change of its semi-axis by an amount dr influences the energy as

$$dE = \frac{GMm}{2r^2} dr \quad (65)$$

Such a change can be brought about by a perturbing force, most likely friction due to gas viscosity in our case.

The energy E_d deposited per unit time by the accreting matter in the region between r and $r + dr$ scales as $E_d \propto dr/r^2$. The energy E_T radiated per unit time, on the other hand, scales with the surface of the emitting region, i.e. is proportional to $rdrT^4$, so

$$\frac{dr}{r^2} \propto rdrT^4 \quad (66)$$

which, integrating, yields

$$T \propto r^{-3/4} \quad (67)$$

so for a face-on disk at distance D from the observer we obtain a spectrum

$$\frac{1}{D^2} \int_{r_i}^{r_o} I(E, T) 2\pi r dr = \frac{8\pi r_i^2}{3D^2 T_i} \int_{T_o}^{T_i} \left(\frac{T}{T_i}\right)^{-11/3} I(E, T) dT \quad (68)$$

where

$$I(E, T) dE = A \frac{E^3 dE}{(e^{E/kT} - 1)} \quad (69)$$

is the Planck distribution, r_i is the innermost radius of the disk, nearest to the accreting object, r_o the outermost radius and T_i and T_o the corresponding temperatures at these radii. Notice that $r_o \gg r_i$ and $T_i > T_o$.

The parameter T_i , which is thus obtained by fitting the spectrum, depends on the mass of the accreting object (Makishima et al. 2000):

$$T_i \propto M^{-1/4} \quad (70)$$

so lower temperatures of the inner edge, correspond to higher masses of the accreting object.

So high-luminosity ULXs with spectra that are well fit by a MCD model with low T_l are good IMBH candidates. Typical temperatures obtained from Galactic X-ray sources associated with stellar-mass accreting objects are in the 0.4 - 1.0 keV range, so the expected temperature for an object ≈ 100 times more massive will range from 0.1 to 0.3 keV . Colbert & Miller (2005) (Table 1) report several ULXs having T_l in this range and a luminosity in excess of 10^{40} erg/s , which are potential IMBH candidates.

3.2.4 ULXs and star-forming regions

As a last remark on ULXs, we notice that they appear to be associated to active star-forming regions (Humphrey et al. 2003; Swartz et al. 2004; Liu & Bregman 2005), in particular interacting galaxy pairs: e.g. the Antennae (NGC 4038/4039) (Fabbiano et al. 2001) and the interacting galaxy pair NGC 4485/4490 (Roberts et al. 2002). If ULXs are really associated to IMBHs this feature would support the runaway merging model of IMBH origin, as young clusters with high-mass main sequence stars are found virtually only in star-forming regions.

3.3 Other phenomena possibly related to IMBHs

Other phenomena have been regarded as possible signs of the presence of an IMBH in a star cluster. Some of them are reviewed in the following, with the caveat that the listing may be incomplete due to the large amount of work published on the subject.

3.3.1 Millisecond pulsar timing

D’Amico et al. (2002) use the timing of three Milli-Second Pulsars (MSPs) in the GC NGC 6752 to obtain lower limits to their line-of-sight acceleration due to the cluster potential. This implies a lower limit of about $10 M_\odot/L_\odot$ for the central mass-to-light ratio, equivalent to $1.3 \times 10^4 M_\odot$ in dark objects within the innermost 0.15 pc around the cluster center. IMBH of this mass would be a solution, but such a central increase in the mass-to-light ratio can be explained away by dark remnants (stellar-mass black holes and neutron stars) which undergo mass-segregation and accumulate in the center of the cluster potential well.

3.3.2 High-velocity stars

Trenti et al. (2007) shows, among other results, that a population of hard binaries interacting with an IMBH can produce several stars which escape the cluster with a velocity of hundreds of km/s , much higher than typical cluster escape velocities. Hills (1988) proposed to use high-velocity stars (possibly) ejected from the galactic center as a test for the presence of a super-massive black hole. Yu & Tremaine (2003) estimates the number and the velocity of high-velocity stars produced by various mechanisms (gravitational encounters of single stars, tidal breakup of a binary star, ejection by a binary black hole) analytically in quantitative detail. In principle these estimates can be adapted to star clusters, yielding a potential method for detecting IMBHs.

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